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AVAILABILITY ENHANCEMENT OF SERIES PARALLEL SYSTEM

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ABSTRACT
This study analyses the performance of a solar panel system consisting of two subsystem, subsystem A consists of four identical units of solar panels arranged in series parallel and two identical units of batteries arranged in series parallel. The subsystem works if at least one solar panel and one battery work. Subsystem B consists of two identical units of battery and two identical units of inverter connected in series parallel. The subsystem works if at least one unit of inverter and one unit of battery work. When any of the units fails, it is immediately required to be repaired. Failure and repair rates are assumed to be exponentially distributed. The subsystems steady state availability, a key reliability metric, were derived and quantitatively evaluated through the application of the Kolmogorov forward equations method. The results demonstrate that system availability is significantly improve by adopting major repair. The finding provides a theoretical basis for predicting system performance and informing maintenance strategies.

Keywords: Availability, Reliability, failure and repair.

1.0 INTRODUCTION
A sophisticated system's reliability, availability, maintainability, and dependability (RAMD) investigation is particularly beneficial in determining potential design changes. These changes are necessary to improve the system's availability, mean time between failures, and reliability. The implementation of the component determines the implementation of the industrial system. In order to apply certain maintenance techniques on the most important component, it is required to identify it. Most businesses employ RAMD indices in order to maintain modesty and provide accurate and timely services. If a company's systems are unreliable, it cannot implement a rapid reaction strategy. As a result, the majority of industrial multinational corporations are pushing for operational maintenance methods. RAMD performs various system analyses. The system's important recital metrics, including MTBF, MTTR, reliability, availability, maintainability, dependence ratio, and dependency minimum values, can be discovered by RAMD analysis. The maintenance manager to plan system maintenance, these results are useful. RAMD is a key quality parameter in system analysis and is regarded as a good place to start when making enhancements to the system. The need for service providers to execute suitable management strategies for these systems to improve their availability and profitability in order to meet the most stringent criteria has continually increased in several industries, along with the development of technological systems. Knowing the RAMD of the key pieces of equipment used in these businesses is one crucial aspect in this regard. Evidently, in these industries, a fault-based (Breakdown) maintenance system is an expensive and time consuming process that causes a significant conceptual loss to grid operators. Clearly, incorporating dependability into a prosecution's reliability, availability, and maintainability. The significant performance measures can be

derived via RAMD evaluation. These measures include MTBF, MTTR, availability, reliability, maintainability and dependability ratio. In order to ensure system reliability and availability, as well as to improve system features, RAMD approach is commonly used by engineers. For analyzing system performance, strength, and efficacy of the system and its components under various scenarios, numerous scientists have previously presented a variety of approaches in the Felds of solar photovoltaic system, solar-powered water pumping system, RAM and RAMD reliability analysis. Sequel to the above assertion, researchers have developed different maintenance models and strategies in enhancing the system performance and optimizing the system RAMD. Few of such are; the work of [1] Reliability Evaluation of Rooftop Solar Photovoltaic Using Coherent Threshold Systems. The Gumbel-Hougaard family Copula was used by [2] to simulate and evaluate the reliability and performance prediction of a small serial solar photovoltaic systems for rural consumptions. Reliability, Availability, Maintainability and Dependability (RAMD) Analysis of Computer Based Test (CBT) Network System by [3]. [4] Studied Performance Evaluation of Complex Reverse Osmosis Machine system in water Purification using Reliability, Availability, Maintainability and Dependability Analysis. [5] Looked at the reliability and performance analysis of a serial parallel photovoltaic system with human operators using Gumbel–Hougaard family copula. Performance modelling and enhancement of solar water pumping system using Gumbel-Hougaard family copula by [6]. Markov chain availability and sensitivity analysis of solar water pumping system by [7]. Research article performance Analysis of tyre manufacturing system in the SMEs using RAM approach carried out by [8]. [9], studied Performance modelling and enhancement of solar water pumping system using Gumbel – Hougaard family copula. The cost analysis of a solar water pumping system for small town's water supply has been looked up by [10]. The Markov chain availability and sensitivity analysis of a solar water pumping system was done by [11].

Despite the growing adoption of solar water systems, a substantial gap remains in the systematic application of availability-based frameworks to evaluate and optimize their performance. Existing studies often focus on technical design or energy efficiency, with less emphasis on how availability modeling can guide proactive repair strategies, reduce downtime, and extend system lifespan. This study seeks to bridge this gap by developing a comprehensive availability-oriented analytical model for solar water systems. Through this framework, the research aims to provide actionable insights into enhancing system performance, improving resilience against uncertainties, and ensuring consistent water supply for rural and agricultural applications.

2.0 NOTATIONS

α_1, α_2 *and* α_3: repair rates of solar panel, battery and inverter respectively.

λ_1, λ_2 *and* λ_3 : failure rates of solar panel, battery and inverter respectively

$B(t)$: Transition probability.

$B_0(t)$: Transition probability that the system is in perfect state with time variable *t*.

$B_1(t)$: Transition probability that the system is in the state 1 with time variable *t*.

$B_2(t)$: Transition probability that the system is in the state 2 with time variable *t*.

$B_3(t)$: Transition probability that the system is in the state 3 with time variable *t*.

$B_4(t)$: Transition probability that the system is in the state 4 with time variable *t*.

$B_5(t)$: Transition probability that the system is in the state 5 with time variable *t*.

$B_6(t)$: Transition probability that the system is in the state 6 with time variable *t*.

$AV_A(\infty)$: *Steady state availability of subsystem A.*

$AV_B(\infty)$:*Steady state availability of subsystem B.*

3.0 AVAILABILITY FORMULATION

3.1 States of subsystem A (Solar panel and Battery)

S_0 : All the units are in good working condition; the subsystem is working

S_1 : One solar panel unit failed, the remaining three solar panels and the two batteries are working; the subsystem is working

S_2 : Another solar panel unit has failed, the remaining two solar panels and two batteries are working, the subsystem is working

S_3 : The third unit of solar panels has failed, the remaining one solar panel and two batteries are working, the subsystem is working

S_4 : The remaining unit of solar panel has failed, but the two batteries are operational; the subsystem is working

S_5 : All the four solar panels have failed and one unit of battery has failed; the subsystem is working.

S_6 : All the four solar panels have failed and two units of battery have failed; the subsystem has failed.

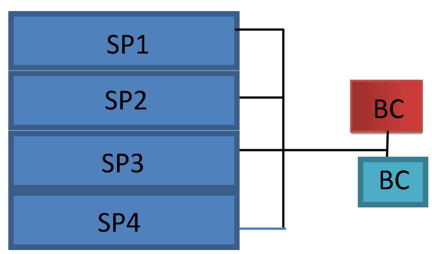
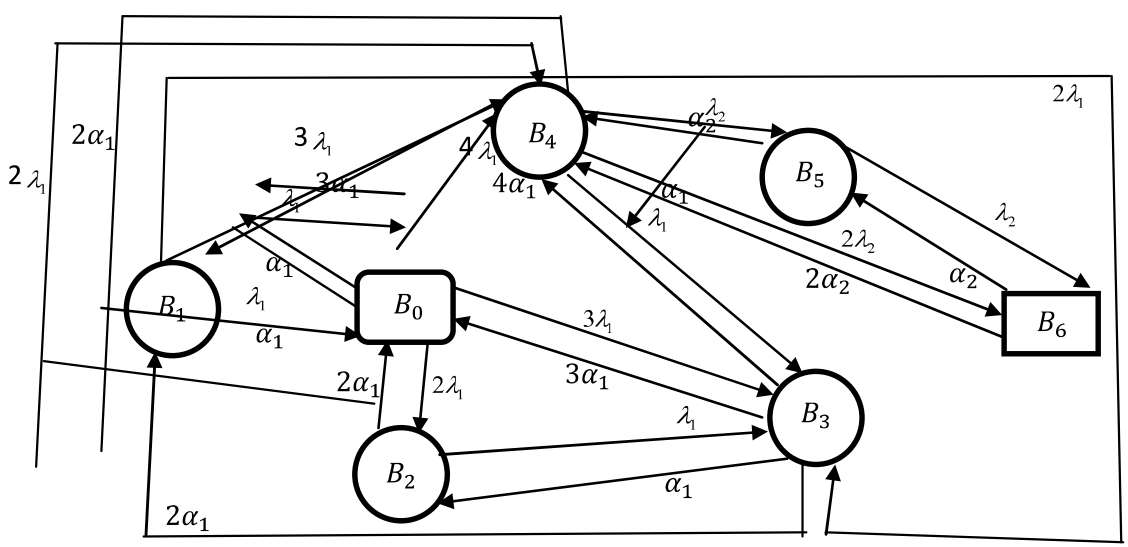


Figure 1. Block Diagram of subsystem A



From Figure 2, system of first order ordinary differential equation is derived as;

$$\frac{dB_0(t)}{dt} = \alpha_1 B_1(t) + 2\alpha_1 B_2(t) + 3\alpha_1 B_3(t) + 4\alpha_1 B_4(t) - 10\lambda_1 B_0(t)$$
$$\frac{dB_1(t)}{dt} = \lambda_1 B_0(t) + \alpha_1 B_2(t) + 2\alpha_1 B_3(t) + 3\alpha_1 B_4(t) - (\alpha_1 + 6\lambda_1) B_1(t)$$
$$\frac{dB_2(t)}{dt} = 2\lambda_1 B_0(t) + \lambda_1 B_1(t) + \alpha_1 B_3(t) + 2\alpha_1 B_4(t) - (3\alpha_1 + 3\lambda_1) B_2(t)$$
$$\frac{dB_3(t)}{dt} = 3\lambda_1 B_0(t) + 2\lambda_1 B_1(t) + \lambda_1 B_2(t) + \alpha_1 B_4(t) - (6\alpha_1 + \lambda_1) B_3(t) \quad (1)$$
$$\frac{dB_4(t)}{dt} = 4\lambda_1 B_0(t) + 3\lambda_1 B_1(t) + 2\lambda_1 B_2(t) + \lambda_1 B_3(t) + \alpha_2 B_4(t) + 2\alpha_2 B_5(t) - (10\alpha_1 + 3\lambda_2) B_6(t)$$
$$\frac{dB_5(t)}{dt} = \alpha_2 B_6(t) + \lambda_2 B_4(t) - (\alpha_2 + \lambda_2) B_5(t)$$
$$\frac{dB_6(t)}{dt} = \lambda_2 B_5(t) + 2\lambda_2 B_4(t) - 3\alpha_2 B_6(t)$$

The above system of differential equations can be expressed into a matrix form as;

$$B^1 = AB \quad (2)$$

where,

$$A = \begin{bmatrix} -10\lambda_1 & \alpha_1 & 2\alpha_1 & 3\alpha_1 & 4\alpha_1 & 0 & 0 \\ \lambda_1 & -(\alpha_1 + 6\lambda_1) & \alpha_1 & 2\alpha_1 & 3\alpha_1 & 0 & 0 \\ 2\lambda_1 & \lambda_1 & -(3\alpha_1 + 3\lambda_1) & \alpha_1 & 2\alpha_1 & 0 & 0 \\ 3\lambda_1 & 2\lambda_1 & \lambda_1 & -(6\alpha_1 + \lambda_1) & \alpha_1 & 0 & 0 \\ 4\lambda_1 & 3\lambda_1 & 2\lambda_1 & \lambda_1 & \alpha_2 & 2\alpha_2 & \mu \\ 0 & 0 & 0 & 0 & \lambda_2 & -(\alpha_2 + \lambda_2) & \alpha_2 \\ 0 & 0 & 0 & 0 & 2\lambda_2 & \lambda_2 & -3\alpha_2 \end{bmatrix},$$

$\mu = -(10\alpha_1 + 3\lambda_2)$,

$B = \left[B_0(t), B_1(t), B_2(t), B_3(t), B_4(t), B_5(t) B_6(t)\right]^T$, $\quad B^1 = \left[B_0^1(t), B_1^1(t), B_2^1(t), B_3^1(t), B_4^1(t), B_4^1(t), B_5^1(t), B_6^1(t)\right]^T$

The initial conditions for the system are;

$$B(0) = [B_0(0), B_1(0), B_2(0), B_3(0), B_4(0), B_5(0), B_6(0)\ ] = [1, 0, 0, 0, 0, 0, 0] \quad (3)$$

The steady-state availability of subsystem A is given by

$$AV_A(\infty) = B_0(\infty) + B_1(\infty) + B_2(\infty) + B_3(\infty) + B_4(\infty) + B_5(\infty) \quad (4)$$

At a steady state, the derivative of the state probabilities becomes zero i.e.

$$\begin{bmatrix} -10\lambda_1 & \alpha_1 & 2\alpha_1 & 3\alpha_1 & 4\alpha_1 & 0 & 0 \\ \lambda_1 & -(\alpha_1+6\lambda_1) & \alpha_1 & 2\alpha_1 & 3\alpha_1 & 0 & 0 \\ 2\lambda_1 & \lambda_1 & -(3\alpha_1+3\lambda_1) & \alpha_1 & 2\alpha_1 & 0 & 0 \\ 3\lambda_1 & 2\lambda_1 & \lambda_1 & -(6\alpha_1+\lambda_1) & \alpha_1 & 0 & 0 \\ 4\lambda_1 & 3\lambda_1 & 2\lambda_1 & \lambda_1 & \alpha_2 & 2\alpha_2 & \mu \\ 0 & 0 & 0 & 0 & \lambda_2 & -(\alpha_2+\lambda_2) & \alpha_2 \\ 0 & 0 & 0 & 0 & 2\lambda_2 & \lambda_2 & -3\alpha_2 \end{bmatrix} \begin{bmatrix} B_0(\infty) \\ B_1(\infty) \\ B_2(\infty) \\ B_3(\infty) \\ B_4(\infty) \\ B_5(\infty) \\ B_6(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{5}$$

Using the normalizing condition

$$B_0(\infty)+B_1(\infty)+B_2(\infty)+B_3(\infty)+B_4(\infty)+B_5(\infty)+B_6(\infty)=1 \tag{6}$$

We substitute (6) in the last row of (5), the resulting matrix will be

$$\begin{bmatrix} -10\lambda_1 & \alpha_1 & 2\alpha_1 & 3\alpha_1 & 4\alpha_1 & 0 & 0 \\ \lambda_1 & -(\alpha_1+6\lambda_1) & \alpha_1 & 2\alpha_1 & 3\alpha_1 & 0 & 0 \\ 2\lambda_1 & \lambda_1 & -(3\alpha_1+3\lambda_1) & \alpha_1 & 2\alpha_1 & 0 & 0 \\ 3\lambda_1 & 2\lambda_1 & \lambda_1 & -(6\alpha_1+\lambda_1) & \alpha_1 & 0 & 0 \\ 4\lambda_1 & 3\lambda_1 & 2\lambda_1 & \lambda_1 & \alpha_2 & 2\alpha_2 & \mu \\ 0 & 0 & 0 & 0 & \lambda_2 & -(\alpha_2+\lambda_2) & \alpha_2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} B_0(\infty) \\ B_1(\infty) \\ B_2(\infty) \\ B_3(\infty) \\ B_4(\infty) \\ B_5(\infty) \\ B_6(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The steady state availability of the subsystem is

$$AV_A(\infty)=\frac{N_A}{D_A} \tag{7}$$

where,

$$\begin{aligned} N_A = {} & (10(180\alpha_1 2\alpha_1^4\lambda_1 + 832\alpha_2\alpha_1^3\lambda_1^2 + 72\alpha_2\lambda_2\alpha_1^3\lambda_1 + 624\alpha_2\alpha_1^2\lambda_1^3 + 384\alpha_2\lambda_2\alpha_1^2\lambda_1^2 + 72\alpha_2\alpha_1\lambda_1^4 \\ & + 332\alpha_2\lambda_2\alpha_1\lambda_1^3 + 45\alpha_2\lambda_2\lambda_1^4)) - (2(135\alpha_1^4\alpha_2^2 + 90\lambda_2\alpha_1^4\alpha_2 + 624\alpha_1^3\alpha_2^2\lambda_1 + 416\lambda_2\alpha_1^3\alpha_2\lambda_1 + \\ & 468\alpha_1^2\alpha_2^2\lambda_1^2 + 312\lambda_2\alpha_1^2\alpha_2\lambda_1^2 + 54\alpha_1\alpha_2^2\lambda_1^3 + 36\lambda_2\alpha_1\alpha_2\lambda_1^3)) + (2(216\alpha_1^3\alpha_2^2\lambda_1 + 144\lambda_2\alpha_1^3\alpha_2\lambda_1 \\ & + 1152\alpha_1^2\alpha_2^2\lambda_1^2 + 768\lambda_2\alpha_1^2\alpha_2\lambda_1^2 + 996\alpha_1\alpha_2^2\lambda_1^3 + 664\lambda_2\alpha_1\alpha_2\lambda_1^3 + 135\alpha_2^2\lambda_1^4 + 90\lambda_2\alpha_2\lambda_1^4)) + \\ & (-63\alpha_1^3\alpha_2^2\lambda_1 - 42\lambda_2\alpha_1^3\alpha_2\lambda_1 + 114\alpha_1^2\alpha_2^2\lambda_1^2 + 76\lambda_2\alpha_1^2\alpha_2\lambda_1^2 + 327\alpha_1\alpha_2^2\lambda_1^3 + 218\lambda_2\lambda_1\alpha_2\lambda_1^3) + \\ & (87\alpha_1^3\alpha_2^2\lambda_1 + 58\lambda_2\alpha_1^3\alpha_2\lambda_1 + 984\alpha_1^2\alpha_2^2\lambda_1^2 + 656\lambda_2\alpha_1^2\alpha_2\lambda_1^2 + 147\alpha_1\alpha_2^2\lambda_1^3 + 98\lambda_2\alpha_1\alpha_2\lambda_1^3) + \\ & (987\alpha_1^3\alpha_2^2\lambda_1 + 658\lambda_2\alpha_1^2\alpha_2\lambda_1 + 954\alpha_1^2\alpha_2^2\lambda_1^2 + 636\lambda_1^2\lambda_2\alpha_1\alpha_1^2 + 117\alpha_1\alpha_1^2\lambda_1^3 + 78\lambda_2\alpha_1\alpha_2\lambda_1^3) \end{aligned}$$

$$\begin{aligned} D_A = {} & (-270\alpha_1^4\alpha_2^2 + 3600\lambda_1\alpha_1^4\alpha_2 - 180\alpha_1^3\alpha_2\lambda_2 + 1800\alpha_1^4\lambda_1\lambda_2 + 195\alpha_1^3\alpha_2^2\lambda_1 + 16640\alpha_1^3\alpha_2\lambda_1^2 + \\ & 1138\alpha_1^3\alpha_2\lambda_1\lambda_2 + 8320\alpha_1^3\lambda_1^2\lambda_2 + 432\alpha_1^3\lambda_1\lambda_2^2 + 3420\alpha_1^2\alpha_2^2\lambda_1^2 + 12480\alpha_1^2\alpha_2\lambda_1^3 + 7656\alpha_1^2\alpha_2\lambda_1^2\lambda_2 \\ & + 6240\alpha_1^2\lambda_2\lambda_1^3 + 2304\alpha_1^2\lambda_1^2\lambda_2^2 + 2475\alpha_2^2\alpha_1\lambda_1^3 + 1440\alpha_1\alpha_2\lambda_1^4 + 6298\alpha_1\alpha_2\lambda_1^3\lambda_2 + 720\alpha_1\lambda_2\lambda_1^4 + \\ & 1992\alpha_1\lambda_2^2\lambda_1^3 + 270\alpha_2^2\lambda_1^4 + 810\alpha_2\lambda_1^4\lambda_2 + 270\lambda_1^4\lambda_2^2) \end{aligned}$$

3.2 States of subsystem B (Battery and Inverter)

S_0 : The two units of battery and two inverters are in good working condition; the subsystem is working

S_1 : One unit of the battery fails; the remaining units of inverter are working; the subsystem is working.

S_2 : One unit of inverter fails and the two units of battery are working; the subsystem is working.

S_3 : One unit of inverter fails, the remaining units of battery and one inverter are working, the subsystem is working.

S_4 : One unit of battery fails, and other units of inverter are in good working condition; the subsystem is not working

S_5 : One unit of inverter fails; the remaining units of battery are in good working condition; the subsystem is not working

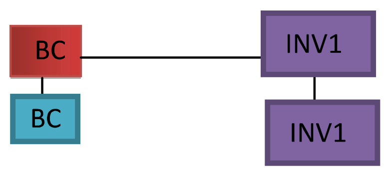


Figure 3. Block Diagram of subsystem B

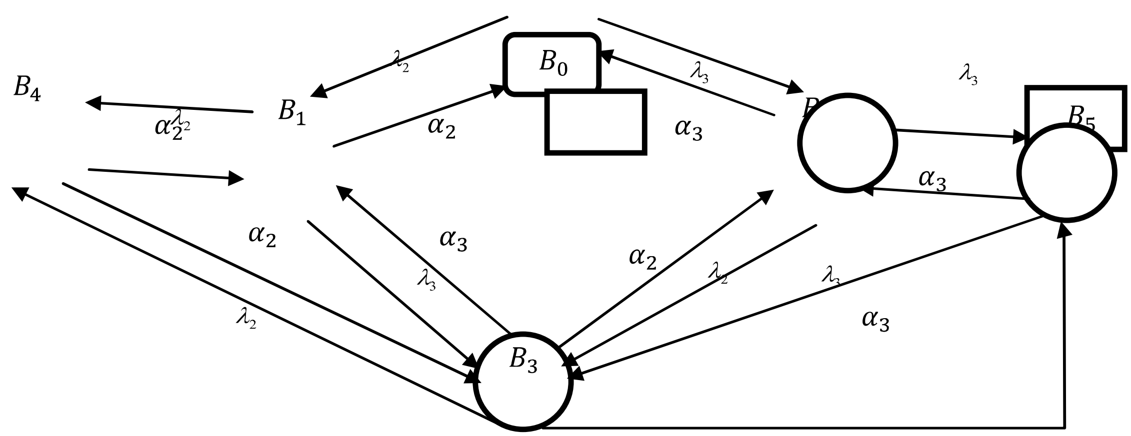


Figure 4. Transition diagram of subsystem B

From Figure 4, system of first order ordinary differential equation is derived as;

$$\frac{dB_0(t)}{dt} = \alpha_2 B_1(t) + \alpha_3 B_2(t) - (\lambda_2 + \lambda_3) B_0(t)$$

$$\frac{dB_1(t)}{dt} = \lambda_2 B_0(t) + \alpha_3 B_3(t) + \alpha_2 B_4(t) - (\alpha_2 + \lambda_2 + \lambda_3) B_1(t)$$

$$\frac{dB_2(t)}{dt} = \lambda_3 B_0(t) + \alpha_2 B_3(t) + \alpha_3 B_5(t) - (\alpha_3 + \lambda_2 + \lambda_3) B_2(t)$$

$$\frac{dB_3(t)}{dt} = \lambda_3 B_1(t) + \lambda_2 B_2(t) + \alpha_2 B_4(t) + \lambda_3 B_5(t) - (2\alpha_3 + \alpha_2 + \lambda_2) B_3(t) \tag{8}$$

$$\frac{dB_4(t)}{dt} = \lambda_2 B_1(t) + \lambda_2 B_3(t) - 2\alpha_2 B_4(t)$$

$$\frac{dB_5(t)}{dt} = \lambda_3 B_2(t) + \alpha_3 B_3(t) - (\alpha_3 + \lambda_3) B_5(t)$$

The above system of differential equations can be expressed into matrix:

$$B^1 = AB \tag{9}$$

$$A = \begin{bmatrix} -(\lambda_2+\lambda_3) & \alpha_2 & \alpha_3 & 0 & 0 & 0 \\ \lambda_2 & -(\alpha_2+\lambda_2+2\lambda_3) & 0 & \alpha_3 & \alpha_2 & 0 \\ \lambda_3 & 0 & -(\alpha_3+\lambda_2+\lambda_3) & \alpha_2 & 0 & \alpha_3 \\ 0 & \lambda_3 & \lambda_2 & -(2\alpha_3+\alpha_2+\lambda_2) & \alpha_2 & \lambda_3 \\ 0 & \lambda_2 & 0 & \lambda_2 & -2\alpha_2 & 0 \\ 0 & 0 & \lambda_3 & \alpha_3 & 0 & -(\alpha_3+\lambda_3) \end{bmatrix}$$

$B = \left[B_0(t), B_1(t), B_2(t), B_3(t), B_4(t), B_5(t)\right]^T$, $\quad B^1 = \left[B_0^1(t), B_1^1(t), B_2^1(t), B_3^1(t), B_4^1(t), B_4^1(t), B_5^1(t),\right]^T$

The initial condition of the subsystem is;

$$B(0) = [B_0(0), B_1(0), B_2(0), B_3(0), B_4(0), B_5(0)] = [1,0,0,0,0,0] \tag{10}$$

The steady-state availability of subsystem B is given by

$$AV_A(\infty) = B_0(\infty) + B_1(\infty) + B_2(\infty) + B_3(\infty) + B_4(\infty) \tag{11}$$

In the steady state, the derivative of the state probabilities becomes zero i.e.

$$A = \begin{bmatrix} -(\lambda_2+\lambda_3) & \alpha_2 & \alpha_3 & 0 & 0 & 0 \\ \lambda_2 & -(\alpha_2+\lambda_2+2\lambda_3) & 0 & \alpha_3 & \alpha_2 & 0 \\ \lambda_3 & 0 & -(\alpha_3+\lambda_2+\lambda_3) & \alpha_2 & 0 & \alpha_3 \\ 0 & \lambda_3 & \lambda_2 & -(2\alpha_3+\alpha_2+\lambda_2) & \alpha_2 & \lambda_3 \\ 0 & \lambda_2 & 0 & \lambda_2 & -2\alpha_2 & 0 \\ 0 & 0 & \lambda_3 & \alpha_3 & 0 & -(\alpha_3+\lambda_3) \end{bmatrix} \begin{bmatrix} B_0(\infty) \\ B_1(\infty) \\ B_2(\infty) \\ B_3(\infty) \\ B_4(\infty) \\ B_5(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{12}$$

The normalizing condition for the subsystem;

$$B_0(\infty) + B_1(\infty) + B_2(\infty) + B_3(\infty) + B_4(\infty) + B_5(\infty) = 1 \tag{13}$$

We substitute (12) in the last row of (11), the resulting matrix will be

$$\begin{bmatrix} -(\lambda_2+\lambda_3) & \alpha_2 & \alpha_3 & 0 & 0 & 0 \\ \lambda_2 & -(\alpha_2+\lambda_2+\lambda_3) & 0 & \alpha_3 & \alpha_2 & 0 \\ \lambda_3 & 0 & -(\alpha_3+\lambda_2+\lambda_3) & \alpha_2 & 0 & \alpha_3 \\ 0 & \lambda_3 & \lambda_2 & -(2\alpha_3+\alpha_2+\lambda_2) & \alpha_2 & \lambda_3 \\ 0 & \lambda_2 & 0 & \lambda_2 & -2\alpha_2 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} B_0(\infty) \\ B_1(\infty) \\ B_2(\infty) \\ B_3(\infty) \\ B_4(\infty) \\ B_5(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The steady state availability of the subsystem is;

$$AV_B(\infty) = \frac{N_B}{D_B} \tag{14}$$

where,

$$N_B = (4\alpha_3^3\lambda_2 + 4\alpha_3^2\lambda_2^2 + 4\alpha_3^2\lambda_2\lambda_3 + \alpha_2\alpha_3^2\lambda_2 + 2\alpha_3\lambda_2^2\lambda_3 + \alpha_2\alpha_3\lambda_2^2 + 4\alpha_3\lambda_2\lambda_3^2 + 4\alpha_2\alpha_3\lambda_2\lambda_3 + 2\alpha_2\alpha_3\lambda_3^3\lambda_2^2\lambda_3$$
$$+ 2\alpha_2\lambda_2\lambda_3^2 + \alpha_2\lambda_3^3) + (-\alpha_2^2\alpha_3\lambda_3 - \alpha_2^2\lambda_2\lambda_3 + 2\alpha_2^2\lambda_3^2 - 3\alpha_2\alpha_3^2\lambda_2 + \alpha_2\alpha_3^2\lambda_3 + 2\alpha_3\alpha_2\lambda_2\lambda_3 + 2\alpha_3\alpha_2\lambda_3^2 +$$
$$2\alpha_2\lambda_2^2\lambda_3 + 4\alpha_2\lambda_2\lambda_3^2 + 2\alpha_2\lambda_3^3 + 4\alpha_3^2\lambda_2^2 + 6\alpha_3^2\lambda_2\lambda_3 + 2\alpha_3^2\lambda_3^3 + 2\alpha_3\lambda_2^3 + 4\alpha_3\lambda_2^2\lambda_3 + 2\alpha_3\lambda_2\lambda_3^2) + (\alpha_2^2\alpha_3\lambda_2$$
$$+ 2\alpha_2^2\alpha_3\lambda_3 + \alpha_2^2\lambda_2\lambda_3 + \alpha_2^2\lambda_3^2\alpha_2\alpha_3^3 + 4\alpha_2\lambda_2\alpha_3^2 + 2\alpha_2\lambda_3\alpha_3^2 + 4\alpha_2\lambda_2\alpha_3\lambda + 4\alpha_2\lambda_3^2\alpha_3 + 4\alpha_3^3\lambda_2 + 2\alpha_3^3\lambda_3 +$$
$$2\alpha_3^2\lambda_2^2 + 2\alpha_3^2\lambda_2\lambda_3) + (2\alpha_3^2\lambda_2\lambda_3 + \alpha_2\alpha_3^2\lambda_2 + 2\alpha_3\lambda_2^3 + 6\alpha_3\lambda_2^2\lambda_3 - \alpha_2\alpha_3\lambda_2^2 + 4\alpha_3\lambda_2\lambda_3^2 + 2\lambda_2^3\lambda_3 + 6\lambda_2^2\lambda_3^2$$
$$- \alpha_2\lambda_2^2\lambda_3 + 6\lambda_2\lambda_3^2 + 2\lambda_3^4 + \alpha_2\lambda_3^3)$$

$$D_B = (-2\alpha_2^2\alpha_3\lambda_2 + 2\alpha_2^2\alpha_3\lambda_3 - 3\alpha_2^2\lambda_3\lambda_2 + 3\alpha_2^2\lambda_3^2 + \alpha_2\alpha_3^3 + \alpha_2\alpha_3^2\lambda_2 + 3\alpha_2\alpha_3^2\lambda_3 - 3\alpha_2\lambda_2^2\alpha_3 + 6\alpha_2\lambda_2\alpha_3\lambda_3 +$$
$$7\alpha_2\lambda_3^2\alpha_3 - 2\alpha_2\lambda_2^3 + 7\alpha_2\lambda_3^2\lambda_2 + 5\alpha_2\lambda_3^2 + 10\alpha_3^3\lambda_2 + 2\alpha_3^3\lambda_3 + 16\alpha_3^2\lambda_2^2 + 19\alpha_3^2\lambda_2\lambda_3 + 2\alpha_3^2\lambda_3^2 + 11\alpha_3\lambda_2^3$$
$$+ 28\alpha_3\lambda_2^2\lambda_3 + 22\alpha_3\lambda_3^2\lambda_2 + 5\alpha_3\lambda_3^3 + 2\lambda_2^4 + 9\lambda_2^3\lambda_3 + 15\lambda_2^2\lambda_3^2 + 11\lambda_3^3\lambda_2 + 3\lambda_3^4)$$

4.0. Results and Discussion

The numerical results pertaining to steady state availability for the two subsystems are presented in this section. the following set of parameters values were used as follow;

$\alpha_2 = 0.1,\ \lambda_1 = 0.4,\ \lambda_2 = 0.5,\ \alpha_3 = 0.3\ \ \lambda_3 = 0.6\ \ for\ Figure\ 4$

$\alpha_1 = 0.1,\ \lambda_1 = 0.4,\ \lambda_2 = 0.5,\ \alpha_3 = 0.3\ \ \lambda_3 = 0.6\ \ for\ Figure\ 5$

$\alpha_1 = 0.1,\ \alpha_2 = 0.2,\ \lambda_2 = 0.5,\ \alpha_3 = 0.3\ \ \lambda_3 = 0.6\ \ for\ Figure\ 6$

$\alpha_1 = 0.1,\ \lambda_1 = 0.4,\ \alpha_2 = 0.2,\ \alpha_3 = 0.3\ \ \lambda_3 = 0.6\ \ for\ Figure\ 7$

$\alpha_1 = 0.1,\ \alpha_2 = 0.2,\ \lambda_2 = 0.5,\ \lambda_1 = 0.4\ \ \lambda_3 = 0.6\ \ for\ Figure\ 8$

$\alpha_1 = 0.1,\ \alpha_2 = 0.15,\ \lambda_2 = 0.5,\ \lambda_1 = 0.4\ \ \alpha_3 = 0.3\ \ for\ Figure\ 9$

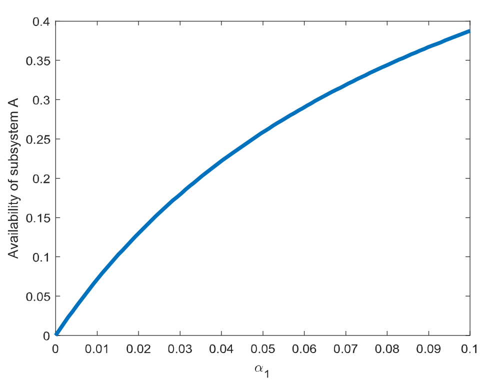


Figure 5. Effect of α_1 on Availability

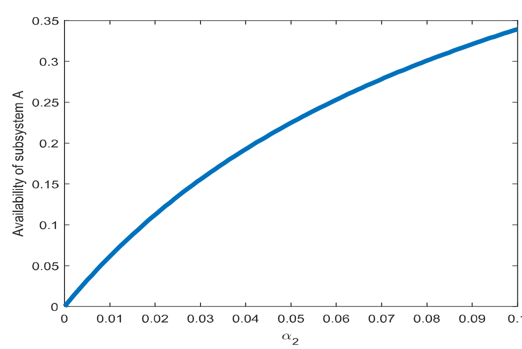


Figure 6. Effect of α_2 on Availability

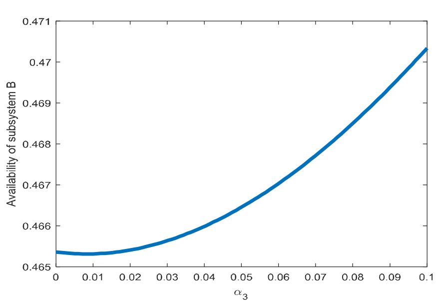


Figure 7. Effect of α_3 on Availability

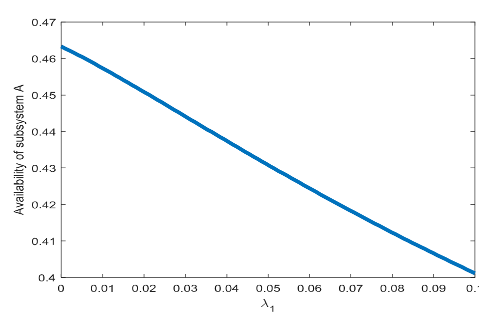


Figure 8. Effect of λ_1 on Availability

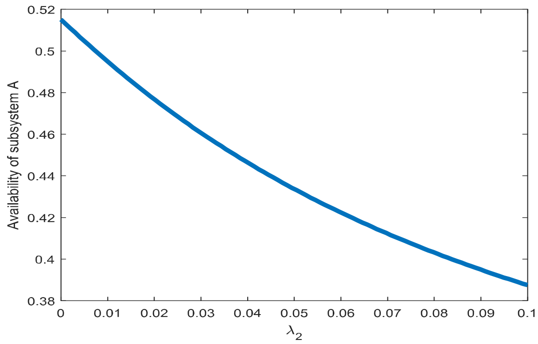


Figure 9. Effect of λ_2 on Availability

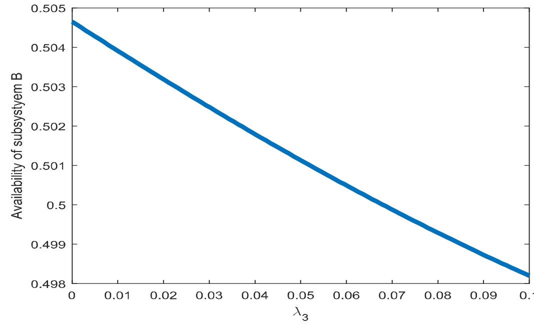


Figure 10. Effect of λ_3 on Availability

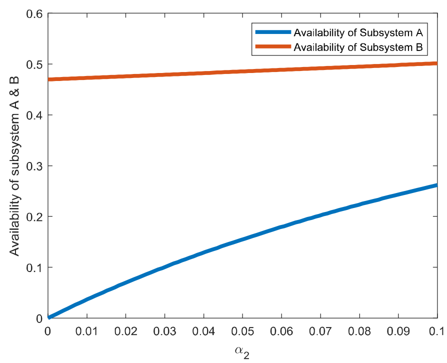


Figure 11. Effect of α_2 on Availability

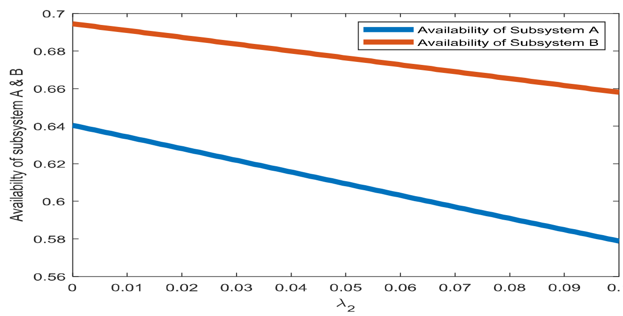


Figure 12. Effect of λ_2 on Availability

The following analysis presents a comprehensive evaluation of system availability, as detailed in Figure 5 through 12. This study specifically investigates the critical interplay between the system's failure rate and repair rate to determine their combined impact on operational availability. The central objective is to identify the condition under which system robustness is maximized. The results desalinate clear and consistent trends revealing how availability respond to change in system reliability and maintainability. The conclusion drawn from each figure collectively build a foundational understanding of the system performance as the results immediately follow:

Analysis of the figures reveals consistent trends in the availability of Subsystems A and B concerning repair and failure rates. For Subsystem A & B: As shown in Figures 5, 6, and 7, the availability of Subsystem A & B increases with the repair rate for all observed values of the parameter α_i. A higher value of α_i = 0.1 corresponds to greater availability, while a lower value of α_i = 0.009 results in diminished availability. This confirms that enhancing the repair rate directly improves system availability. Conversely, Figures 8, 9 and 10 demonstrate that the availability of Subsystem A & B decreases as the failure rate increases. In this scenario, a lower value of λ_i = 0.009 yields higher availability, whereas λ_i = 0.1 leads to lower availability. This indicates that reducing the failure rate is key to improving availability.

The comparison between subsystem A and subsystem B in Figure 11 and 12 shows that, their steady state availability would include that; subsystem B is the primary candidate for redesign or improvement to boost overall system availability, while subsystem A is performing excellently.

4.1 SUMMARY

The analysis of the subsystem steady state availability reveals two consistent, inverse relationships: availability of subsystem A & B decreases as the failure rate increases, and availability increases as the repair rate increases. This trend is robust, observed across the subsystems under a wide range of parameters. The key finding is that subsystem steady state availability is maximized when the failure rate is minimized and the repair rate is maximized. These results provide a clear quantitative foundation for recommending strategies focused on preventive maintenance (to reduce failure rates) and efficient repair processes (to increase repair rates) to enhance overall subsystem steady state reliability.

4.2. CONCLUSION

This study considered a series parallel system consists of two subsystems A and B. each subsystem has two units in series parallel. An explicit expression for the subsystems steady state availability was obtained through Kolmogorov forward equations. Results are displayed in Figure 4 to 9. From the figures it is evident that availability is improved with major repair (higher). The analysis indicates superior performance in subsystem A compare to subsystem B.

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